

SCIENTIFIC PROGRAM

CIMPA ON-LINE PRE-SCHOOL & RESEARCH SCHOOL ALGEBRAIC METHODS IN TOPOLOGY

El Salvador, December 14-19, 2020 & July 11-22, 2022.

PRE-SCHOOL ON-LINE COURSES

Course 1: Foundations of Homotopy Theory

By *Ingrid Martínez*, Universidad de El Salvador.

LANGUAGE: Spanish. FORMAT: on-line course. DURATION: 6 hours.

ABSTRACT: El minicurso de fundamentos de homotopía se desarrollará en tres sesiones donde se abordarán conceptos y resultados importantes de la teoría de homotopía:

Sesión 1: Homotopía de funciones y grupo fundamental. H- espacios, espacios lazo y suspensiones.

Sesión 2: Construcción de espacios: Cilindros y Conos. Grupos de homotopía y sucesiones de homotopía.

Sesión 3: Propiedades de extensión y levantamiento de homotopía.

BIBLIOGRAPHY: *Algebraic Topology from Homotopical Viewpoint*; Aguilar M., Gliter S. & Prieto C.

Course 2: Introduction to Abelian Groups and Modules

By *Gabriel Chicas*, Universidad de El Salvador.

LANGUAGE: Spanish. FORMAT: on-line course. DURATION: 6 hours.

ABSTRACT: En este minicurso abordaremos elementos básicos de la teoría de grupos abelianos y módulos. Nos enfocaremos en algunos problemas de clasificación: el de módulos finitamente generados sobre un dominio de ideales principales, y más generalmente sobre un dominio de Dedekind. Concluiremos con una discusión acerca de módulos proyectivos sobre un dominio de Dedekind.

BIBLIOGRAPHY: *Abstract Algebra, 3rd*; Dummit and Foote. *A Course in Homological Algebra, 2nd*; Hilton and Stambach.

Course 3: An introduction to Stable Homotopy Theory

By *Omar Antolín*, Universidad Nacional Autónoma de México.

LANGUAGE: Spanish. FORMAT: on-line course. DURATION: 6 hours.

ABSTRACT: Los espectros son una manera concreta de representar teorías de cohomología generalizadas en términos de espacios topológicos y tienen una peculiar naturaleza dual, topológica y algebraica. En este mini-curso daremos una introducción a la teoría de homotopía estable, que es la rama de la topología encargada de estudiar a los espectros. Hablaremos de los teoremas de la teoría de homotopía que son precursores de la teoría estable, de las definiciones de espectros y algunos ejemplos clave, y de cómo su estudio es una generalización del álgebra.

BIBLIOGRAPHY: *Stable Homotopy and Generalised Homology*; Adams, Frank; libro clásico, la parte III del libro es la más relevante. *The stable homotopy category*; Malkiewich, Cary; artículo expositivo disponible en la página web del autor: <https://people.math.binghamton.edu/malkiewich/stable.pdf>. *Stable algebraic topology, 1945–1966*; May, Peter; artículo sobre la historia de un periodo muy fértil en la teoría de homotopía estable, disponible en la página web del autor: <http://www.math.uchicago.edu/~may/PAPERS/history.pdf>.

Course 4: Foundations of Module Theory

By *Keri Sather-Wagstaff*, Clemson University.

LANGUAGE: English. FORMAT: on-line course. DURATION: 6 hours.

ABSTRACT: One way to understand a geometric object is to understand its symmetries. This idea translates powerfully to numerous areas of mathematics. In algebra, one way this manifests is through the study of modules: one understands a ring (for instance, arising in geometry or number theory) by understanding the objects it influences. (Maybe think of this like the discovery of Pluto, which was first inferred by astronomers observing *something* affecting other celestial bodies.) In this mini-course, we will present foundational material about rings and modules, in preparation for the follow-up course “Foundations of homological algebra applied to modules.”

BIBLIOGRAPHY: *Abstract Algebra, 3ed*; Dummit and Foote. *Algebra*; Hungerford.

SCHOOL COURSES

Course 1: Topology of function spaces

By *Ingrid Martínez* and *Dimas Tejada*, Universidad de El Salvador.

LANGUAGE: Spanish. FORMAT: classroom course. DURATION: 6 hours.

ABSTRACT: Starting at a basic level, for maps between topological spaces the compact and open topology will be analyzed as well as the exponential law. The concepts of smooth manifolds and maps will be introduced, and the main results on the Weak and Strong Topologies on the Space of C^r -Maps between manifolds will be described. Finally, we will introduce the degree modulo 2 and the Brouwer degree of smooth maps. The course will finish by discussing the celebrated Poincaré-Hopf Theorem.

BIBLIOGRAPHY: *Differential Topology*; Hirsch, Morris. *An introduction to differentiable manifolds and Riemannian Geometry*; Boothby, William.

Course 2: An introduction to Geometric Group Theory

By *Luis Sánchez*, Universidad Nacional Autónoma de México.

LANGUAGE: Spanish. FORMAT: classroom course. DURATION: 6 hours.

ABSTRACT: The main goal of this mini course is to provide an introduction to geometric group theory, that is, the study of groups via their actions on geometric and topological objects. Among the topics we will define and study are: the word metric, the Cayley graph of a group, quasi-isometries, the Schwartz-Minor lemma, and a brief introduction to some quasi-isometry invariants of groups.

BIBLIOGRAPHY: *Geometric group theory, an introduction*; Clara Löh. *Office hours with a geometric group theorist*; ed. by Matt Clay & Dan Margalit.

Course 3: Categories and Modules

By *Nadia Romero*, Universidad de Guanajuato.

LANGUAGE: Spanish. FORMAT: on-line course. DURATION: 6 hours.

ABSTRACT: We will start by recalling the basic notions and results of category theory, in particular those concerning the category of modules over a ring. Then we will apply these results to the theory of representations of finite groups and finally we will see some recent results in representation theory that make use of these categorical methods.

BIBLIOGRAPHY: *Rings and categories of modules*; Anderson, F. y Fuller, K. *Toposes, triples and theories*; Barr M. y Wells C. *Categories for the working mathematician*; Mac Lane, S. *Advanced modern algebra*; Rotman, J.

Course 4: Equivariant Co-Homology

By Noé Bárcenas, Universidad Nacional Autónoma de México.

LANGUAGE: Spanish. FORMAT: classroom course. DURATION: 6 hours.

ABSTRACT: The symmetry of topological objects is reflected in algebraic invariants of co-homological type. In this course we will address the definition and interaction of different versions of equivariant co-homology, and we will establish relations with aspects of algebra, group theory, and homotopy theory.

BIBLIOGRAPHY: *Cohomology of groups*; Brown, Kenneth. *Introductory notes on equivariant cohomology*; Tu, Loring. *Equivariant cohomology theories*; Bredon, Glen.

Course 5: The ∞ -categorical point of view for Thom spectra

By Omar Antolín, Universidad Nacional Autónoma de México.

LANGUAGE: Spanish. FORMAT: classroom course. DURATION: 6 hours.

ABSTRACT: I will describe the ∞ -categorical approach developed by Ando, Blumberg, Gepner, Hopkins, and Rezk to Thom spectra of stable spherical fibrations. I will also mention joint work with Tobias Barthel that uses this approach to study the multiplicative structure of such spectra. This multiplicative structure is characterized by a universal property, which is more useful than might seem given its simplicity. As an application we will discuss E_n -orientations and the Thom isomorphism.

I won't assume familiarity with either ∞ -categories or with E_n -ring spectra; on the contrary, I hope this mini course will serve as an introduction to these topics.

Course 6: Foundations of Homological Algebra applied to Modules

By *Keri Sather-Wagstaff*, Clemson University.

LANGUAGE: English. FORMAT: on-line course. DURATION: 4.5 hours.

ABSTRACT: Each module M over a commutative local Noetherian ring R comes equipped with certain numerical invariants, defined homologically, that allow us to detect certain structural information about the module. For example, the projective dimension detects how close M is to being projective. Not only that, but projective dimension of modules can detect whether a ring is regular, as Auslander, Buchsbaum, and Serre proved in the 1950s, and this led to the solution of a famous open question about the localization properties for regular rings. In this course, I will discuss some of these ideas (there are many out there) focusing on a few favorites, including foundational topics.

BIBLIOGRAPHY: *Abstract Algebra, 3ed*; Dummit and Foote (sections 7.1-9.5, 10.1-11.4, 12.1, 13.1-13.2, and pp. 656-657, 706-709, 714-715, 718-719. *Algebra*; Hungerford (Chapters 1-2 and sections V.1, VII.1, VIII.1, and Theorem VIII.4.9). *Introduction to Commutative Algebra*; Atiyah and MacDonald (Chapters 1-3 and 6).

Course 7: Algebraic K -Theory and connections to Topology

By *Mohamed Elhamdadi*, University of South Florida.

LANGUAGE: English. FORMAT: on-line course. DURATION: 4.5 hours.

ABSTRACT: We will give a foundational course on algebraic K -Theory. We will introduce the K -groups starting from the Grothendieck group K_0 to higher K -groups including the Plus-construction of Quillen. Connections to Hochschild and cyclic homology will be given. If time allows we will discuss a variation of K -theory called Karoubi's K -theory.

BIBLIOGRAPHY: *The K-book. An introduction to algebraic K-theory*; Weibel, Charles. *Cyclic homology*; Loday, Jean-Louis. *Algebraic K-theory and its applications*; Rosenberg, Jonathan. *Approach to algebraic K-theory. Research Notes in Mathematics, 56*; Berrick, A. Jon.

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