AJUSTE DE CURVAS PARA DATOS DE EPIDEMIAS USANDO REGRESIÓN LOGÍSTICA GENERALIZADA

CURVE FITTING FOR EPIDEMIC DATA USING GENERALIZED LOGISTIC REGRESSION

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MATEMÁTICA Y SU DIDÁCTICA 2025

- *AJUSTE DE CURVAS PARA DATOS DE EPIDEMIAS USANDO REGRESIÓN LOGÍSTICA GENERALIZADA*

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  TICOVID GROUP
  COSTA RICA

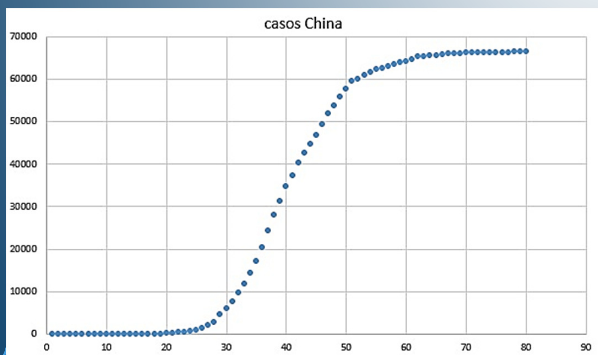
  TICOVID-19

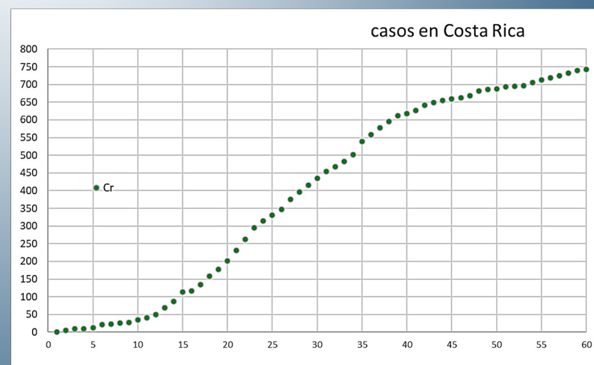
  LAMP TEST to detect covid

  Pharmacy, Biology, Communication, Computing, Statistics, Mathematics, …

POPULATION GROWTH CURVES

- The population growth curves follow the well-known logistic behavior





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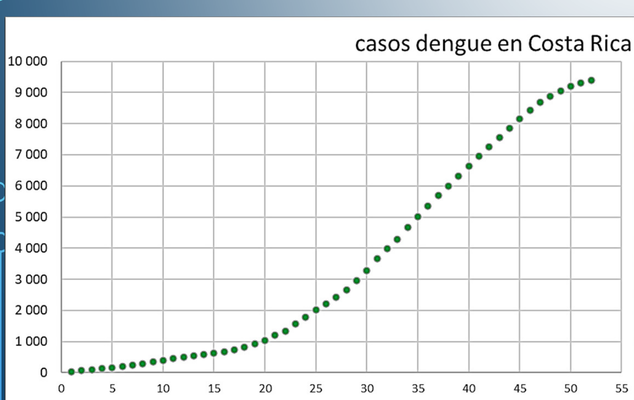

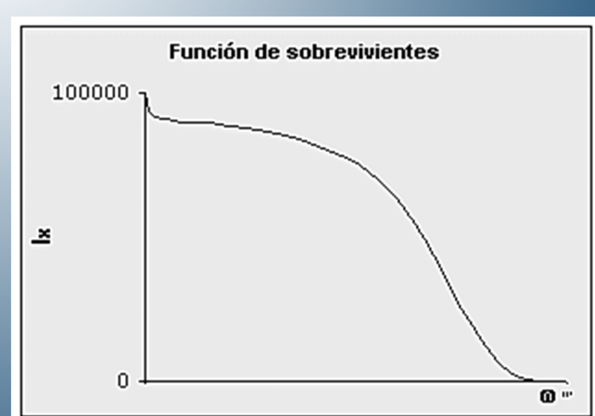

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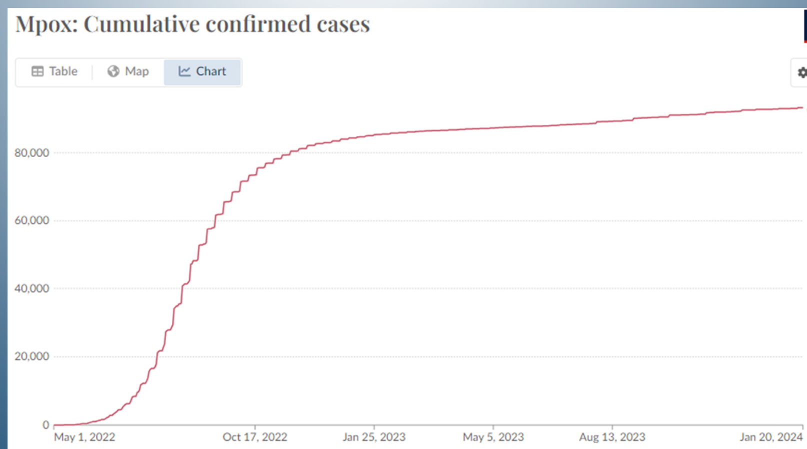

POPULATION GROWTH CURVES

Logistics Curve

$$P(t) = \frac{1}{1 + e^{-at+b}}$$

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Logistics Curve

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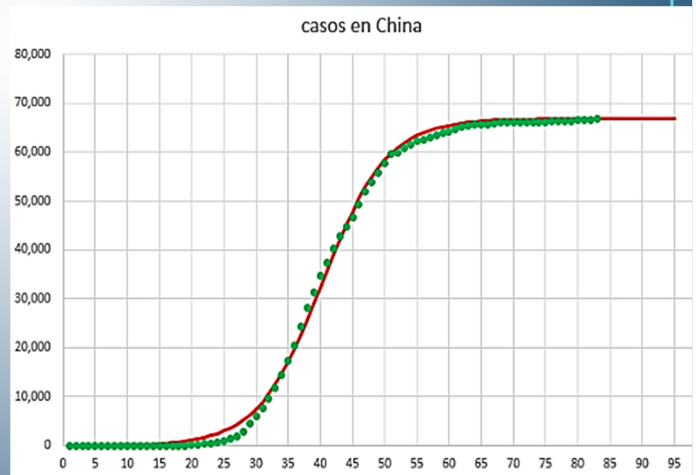


Solution:

- transform the data
- and use linear regression.
- Values in [0,1], not very flexible

CLASICAL MODELS

- **SIR model**

$$\frac{dS}{dt} = -\frac{\beta IS}{N},$$

$$\frac{dI}{dt} = \frac{\beta IS}{N} - \gamma I,$$

$$\frac{dR}{dt} = \gamma I,$$

$$\boxed{S} \xrightarrow{\beta} \boxed{I} \xrightarrow{\gamma} \boxed{R}$$

$$\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0,$$

$$S(t) + I(t) + R(t) = \text{constant} = N,$$

$$\rho_0 = \frac{\beta}{\gamma}$$

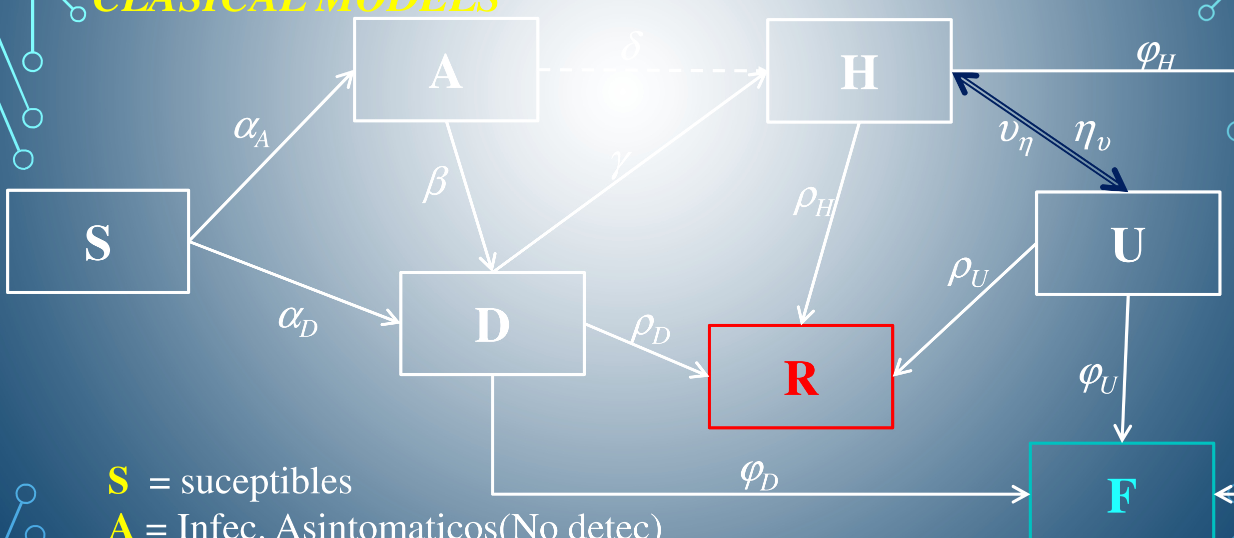
CLASICAL MODELS

- **SIER model**

$$S \xrightarrow{\beta} E \xrightarrow{\alpha} I \xrightarrow{\gamma} R$$

$$\frac{dS}{dt} = -\frac{\beta I S}{N}$$
$$\frac{dE}{dt} = \frac{\beta I S}{N} - \alpha E$$
$$\frac{dI}{dt} = \alpha E - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

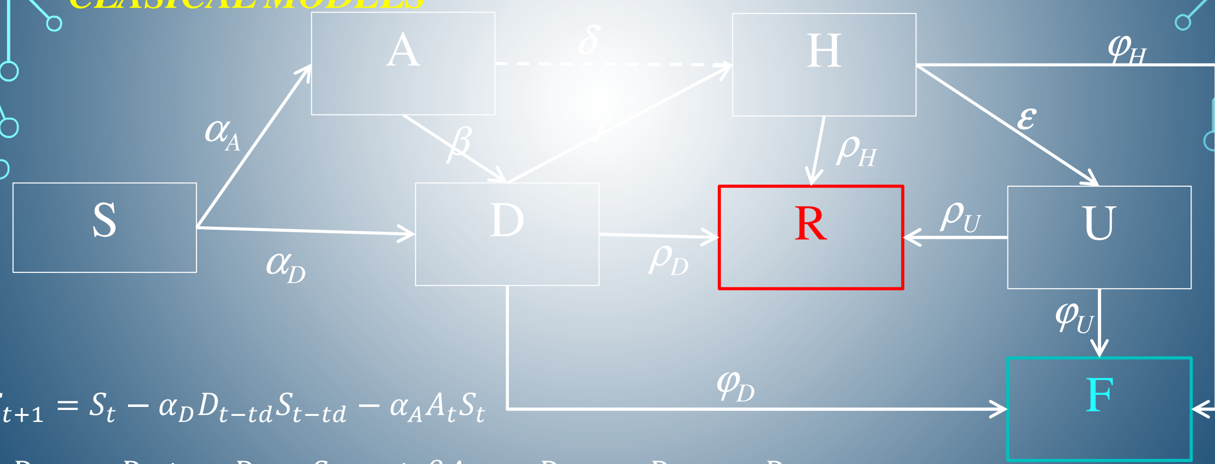
CLASICAL MODELS



S = suceptibles
A = Infec. Asintomaticos(No detec)
D = Infectados Detectados
H = Infec. Hospitalizados
U = Hosp. en UCI

R = Recuperados
F = Fallecidos

CLASICAL MODELS



$$S_{t+1} = S_t - \alpha_D D_{t-td} S_{t-td} - \alpha_A A_t S_t$$

$$D_{t+1} = D_t + \alpha_D D_{t-td} S_{t-td} + \beta A_t - \gamma D_t - \rho_D D_t - \varphi_D D_t$$

$$A_{t+1} = A_t + \alpha_A A_t S_t - \beta A_t - \delta A_t$$

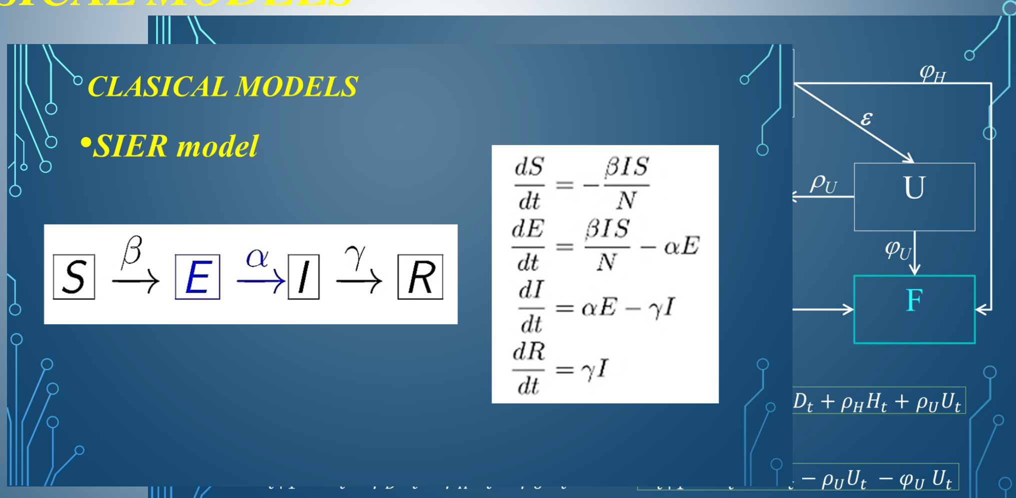
$$F_{t+1} = F_t + \varphi_D D_t + \rho_H H_t + \rho_U U_t$$

$$H_{t+1} = H_t + \gamma D_t + \delta A_t - \varepsilon H_t - \rho_H H_t - \varphi_H H_t$$

$$R_{t+1} = R_t + \rho_D D_t + \rho_H H_t + \rho_U H_t$$

$$U_{t+1} = U_t + \varepsilon H_t - \rho_U U_t - \varphi_U U_t$$

CLASICAL MODELS



El problema con estos $\rightarrow$ los parámetros

con pocos datos $\rightarrow$ difícil determinar

GENERALIZED LOGISTICS REGRESSION

Which "curve"

First proposal

$$P(t) = \frac{M}{1 + e^{-at+b}}$$

More complex

$$P(t) = \frac{M}{(1 + e^{-at+b})^{\alpha}}$$

$\ldots, x^{1/3}, x^{1/2}, x, x^2, x^3, \ldots$

GENERALIZED LOGISTICS REGRESSION

First proposal

$$P(t) = \frac{M}{(1 + e^{-at+b})^{\alpha}}$$

same model as

Richards curve

$$Y(t) = A + \frac{K - A}{(C + Qe^{-Bt})^{1/\nu}}$$

Population growth

$$Y'(t) = \alpha \left(1 - \left(\frac{Y}{K}\right)^{\nu}\right) Y$$

GENERALIZED LOGISTICS REGRESSION

Gompertz Funtion

$$G(t) = ae^{-be^{-ct}}$$

Linear Trend

$$P(t) = \frac{M + ct}{(1 + e^{-at+b})^{\alpha}}$$

Other more complex versions

$$G(t) = (a + dt)e^{-be^{-ct}}$$

$$P(t) = \frac{M}{(1 + e^{-at^2+bt+c})^{\alpha}}$$

$$P(t) = \frac{M + dt}{(1 + e^{-at^2+bt+c})^{\alpha}}$$

GENERALIZED LOGISTICS REGRESSION

Mínimos cuadrados

$$\min_{f} \sum_{t=1}^{N} (d_t - f(t))^2$$

t es el día, d_t es el total de casos acumulados el día *t*,

f(t) es el valor de la función en el día *t*

y utilizamos un algoritmo de optimización No lineal.

En este caso el "solver" de Excel.

(Bueno, con un poco de ayuda)

GENERALIZED LOGISTICS REGRESSION

Hipótesis y propuesta

La propuesta es, utilizar la curva logística generalizada o la curva de Gompertz para hacer un ajuste de los datos.

Hypothesis and proposal

The proposal is to use the generalized logistic curve or the Gompertz curve to make a fit of the data.

GENERALIZED LOGISTICS REGRESSION

Hipótesis y propuesta

Si se tiene la parte bajo de la Curva, (valores iniciales)
¿se puede obtener la curva completa?

Y obtener la tendencia del crecimiento de la población

Hypothesis and proposal

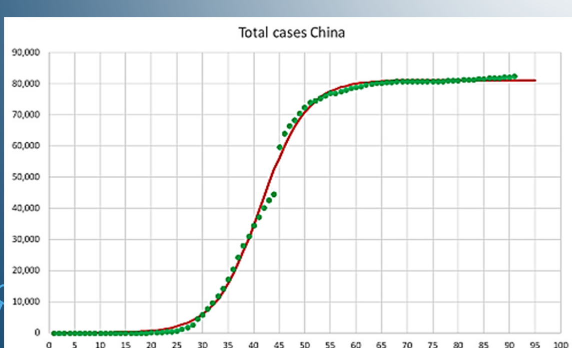
If you have the lower part of the Curve, (initial values)
Can you get the full curve?
And get the trend of population growth

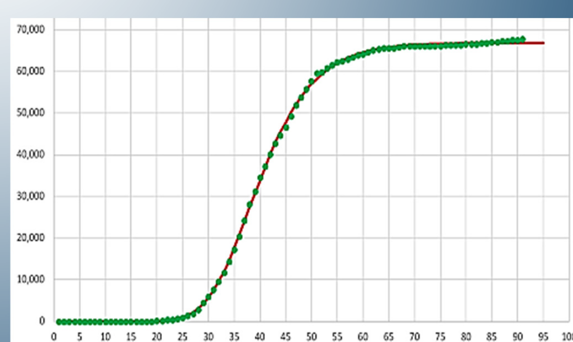
GENERALIZED LOGISTICS REGRESSION

Results

CURVE FITTING

China:

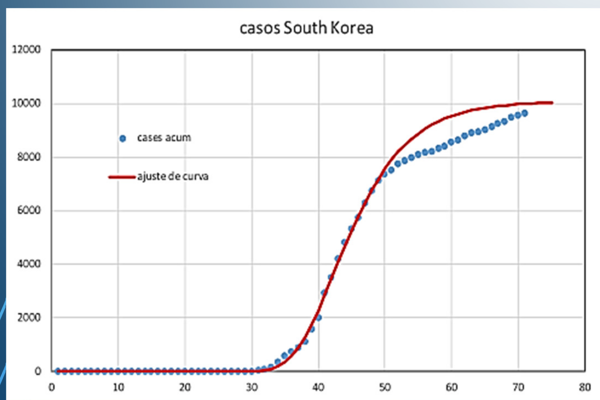


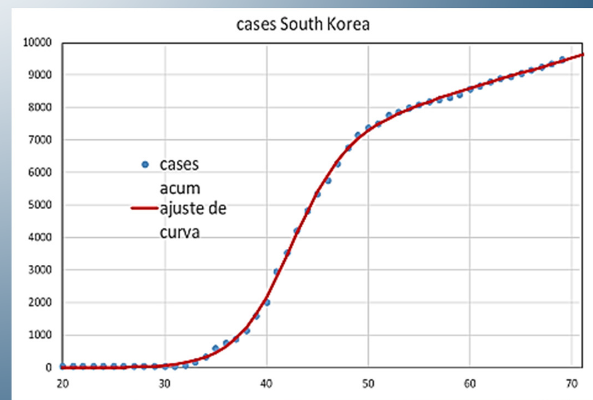


CURVE FITTING

$$P(t) = \frac{M + ct}{(1 + e^{-at+b})^{\alpha}}$$

Corea del Sur





GENERALIZED LOGISTICS REGRESSION

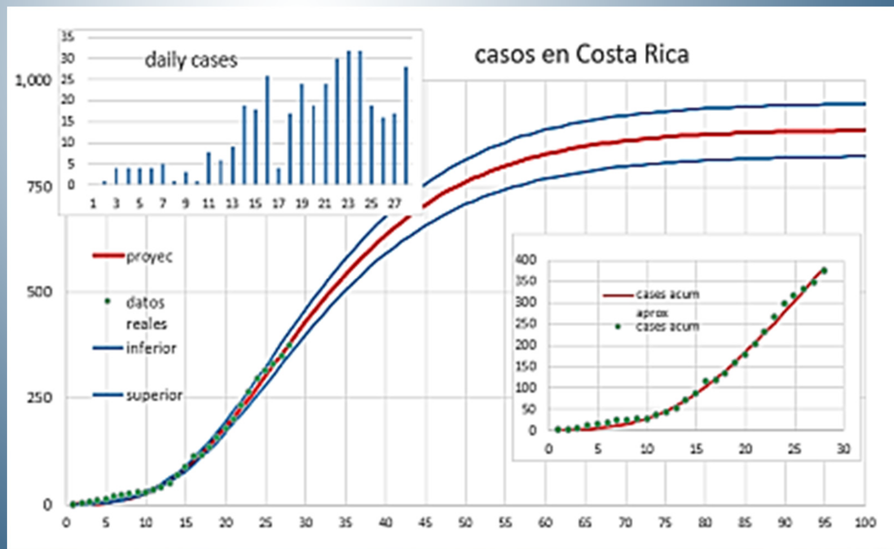
Resultados

Tendencias

PRONÓSTICOS

Costa Rica

LG



FORECASTS OR TRENDS

México

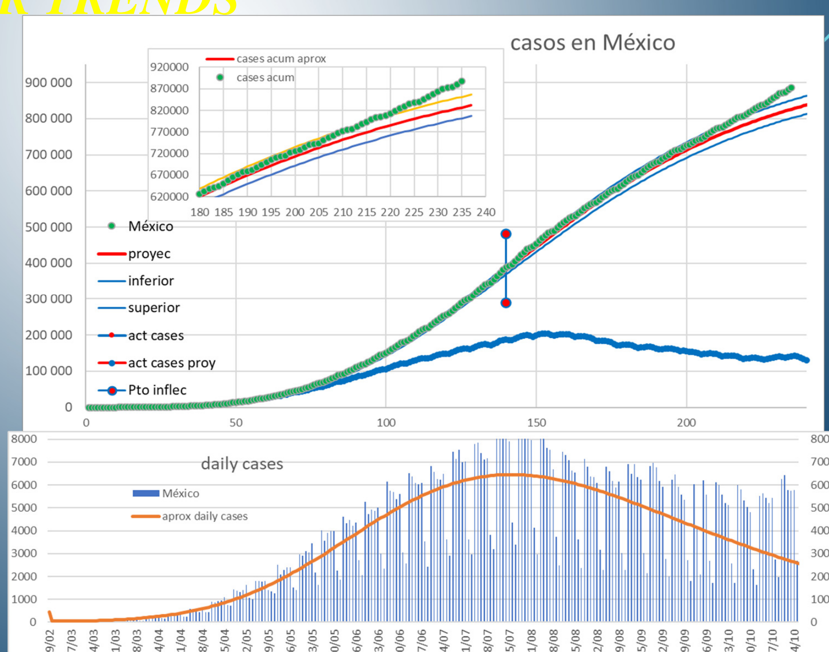
LG –Agosto, 14

and continues

(30-set)

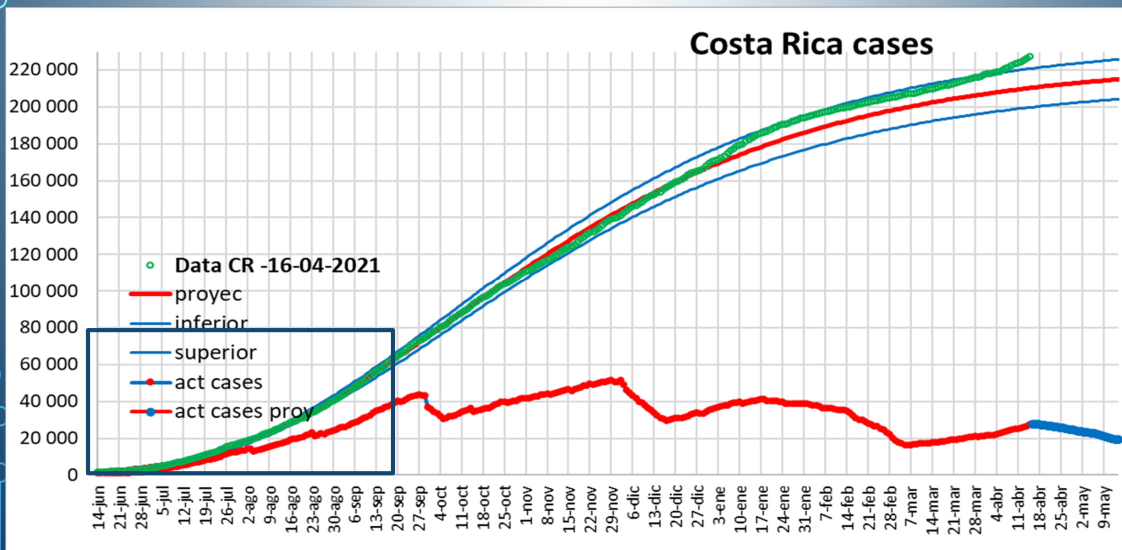
Fit 736 885

Wdm 738,163



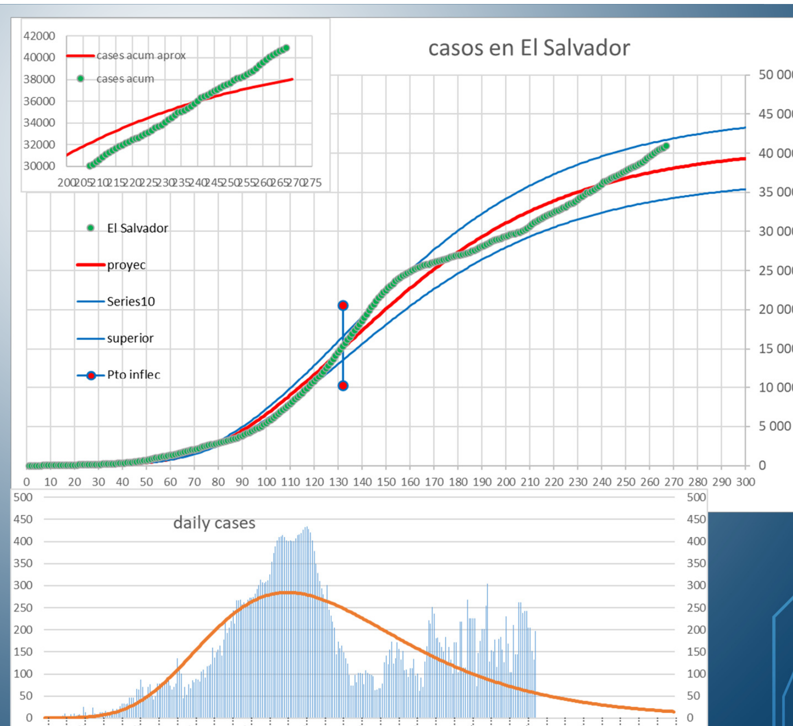
FORECASTS OR TRENDS

Costa Rica LG–September, 19



FORECASTS OR TRENDS

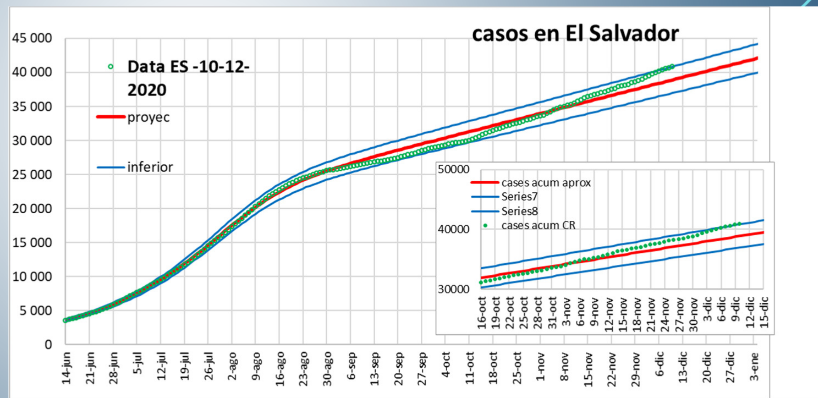
El Salvador

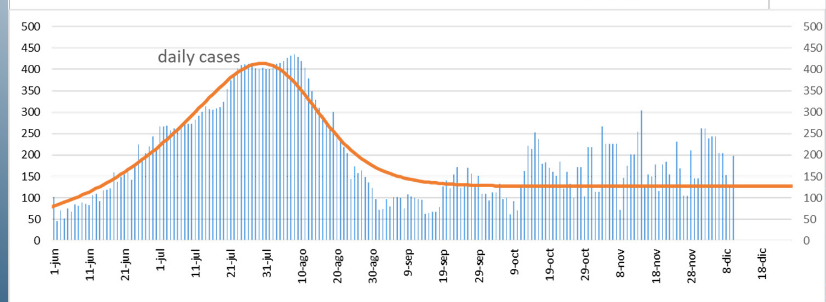


FORECASTS OR TRENDS

El Salvador

Lineal Trend





SECOND WAVES AND MORE

Proposal

$$P_i(t) = \frac{M_i}{(1 + e^{-a_i t + b_i})^{\alpha_i}}$$

$$P(t) = P_0(t) + P_1(t - t_1) + P_2(t - t_2) + \cdots + P_n(t - t_n)$$

Con $0 < t_1 < t_2 < \cdots < t_n$

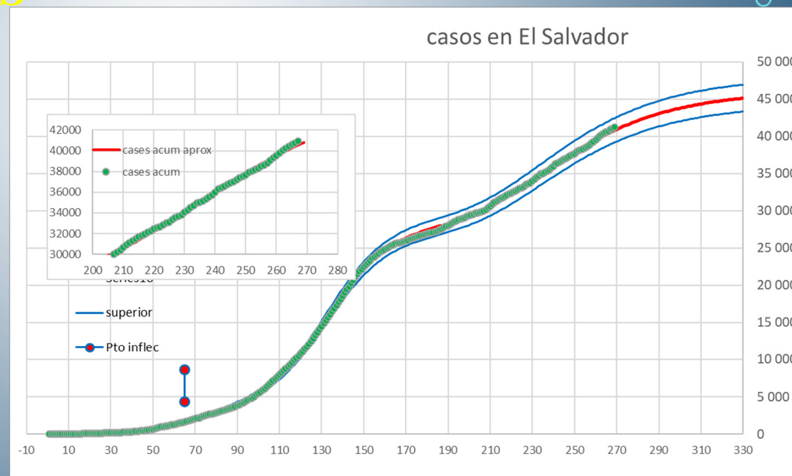
$$G_i(t) = a_i e^{-bie^{-cit}}$$

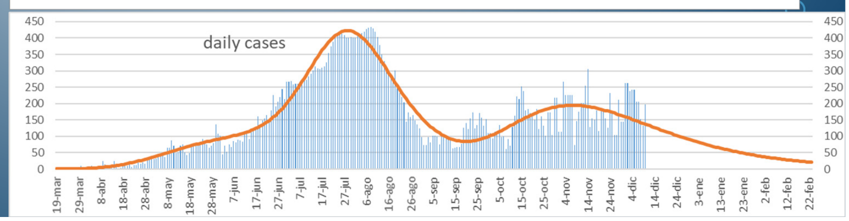
SECOND WAVES AND MORE

El Salvador

$$P(t) = P_0(t) + P_1(t - t_1) + P_2(t - t_2) + \cdots + P_n(t - t_n)$$

$$P_i(t) = \frac{M_i}{(1 + e^{-a_i t + b_i})^{\alpha_i}}$$





SECOND WAVES AND MORE

El Salvador

$$P(t) = P_0(t) + P_1(t - t_1) + P_2(t - t_2) + \cdots + P_n(t - t_n)$$

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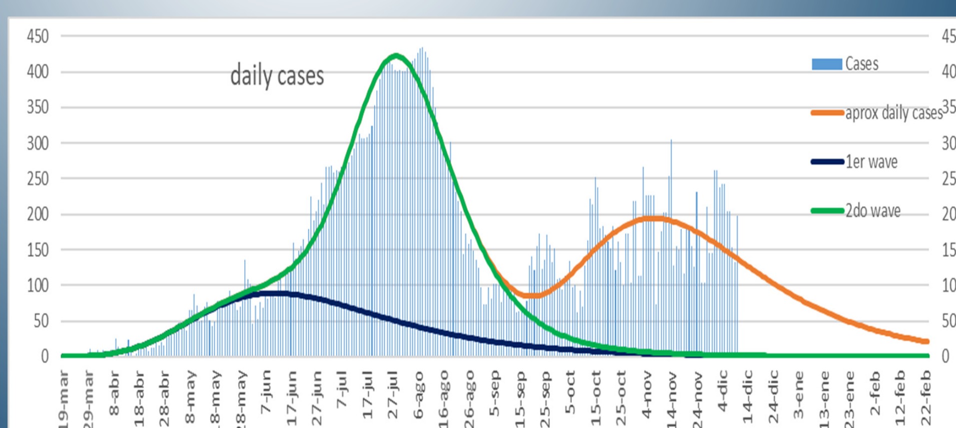
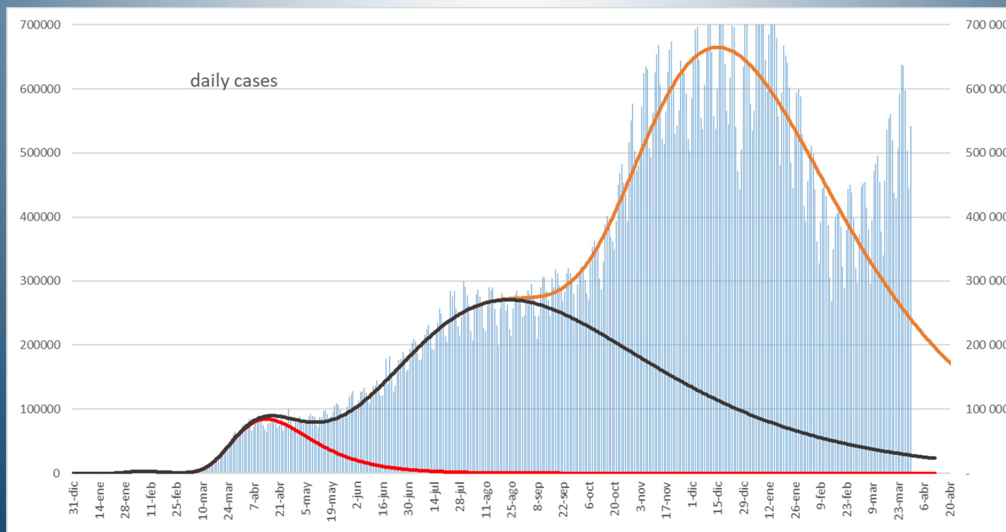


SECOND WAVES AND MORE

$$P_i(t) = \frac{M_i}{(1 + e^{-a_i t + b_i})^{\alpha_i}}$$

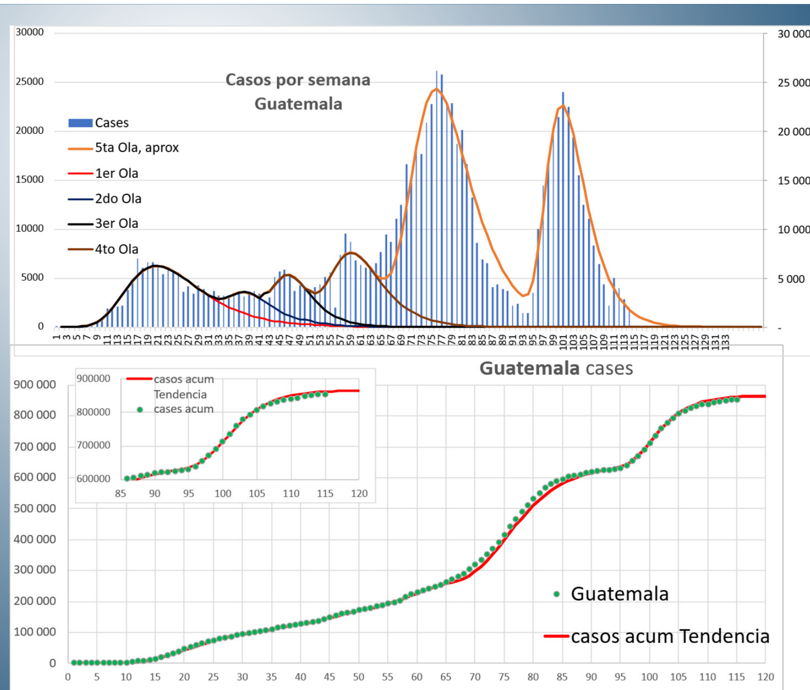
World data

$$P(t) = P_0(t) + P_1(t - t_1) + P_2(t - t_2) + \cdots + P_n(t - t_n)$$



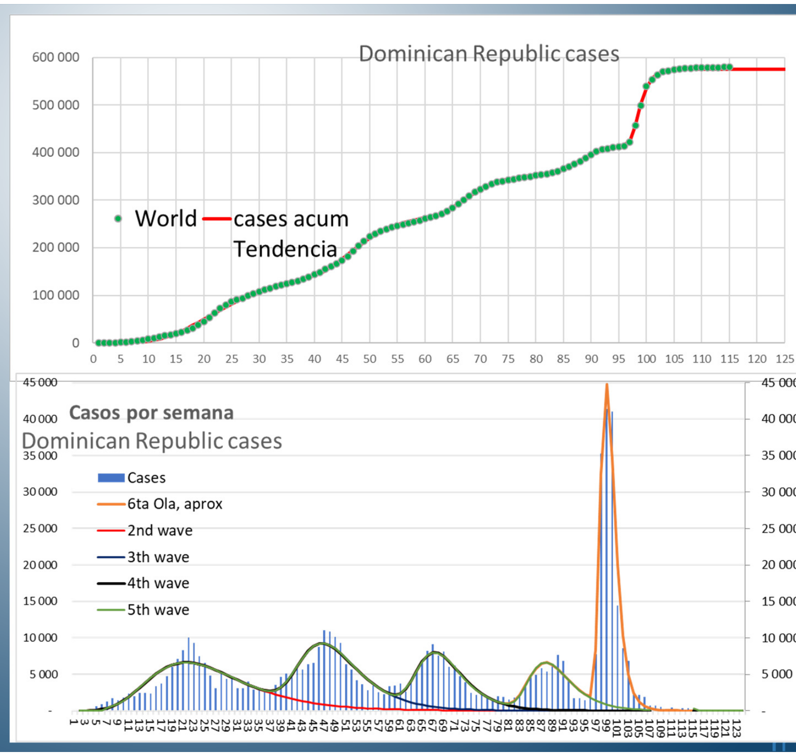
SECOND WAVES AND MORE

Guatemala



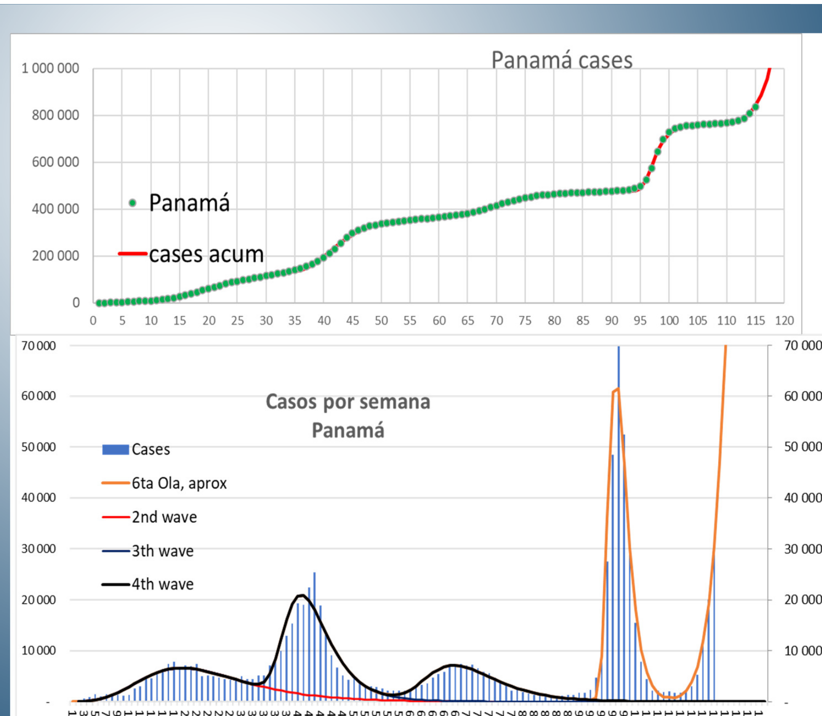
SECOND WAVES AND MORE

República Dominicana



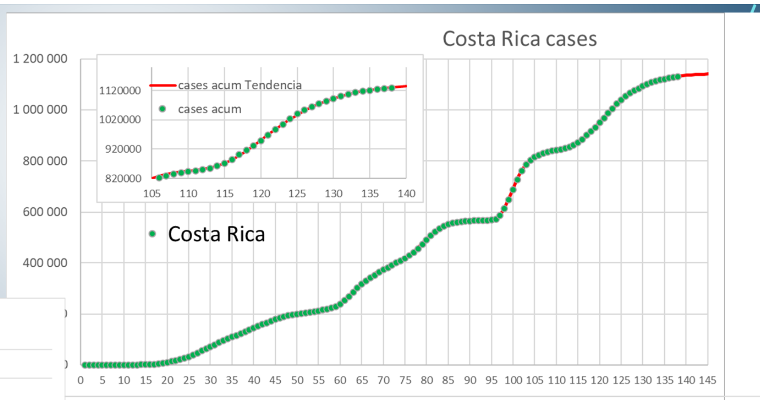
SECOND WAVES AND MORE

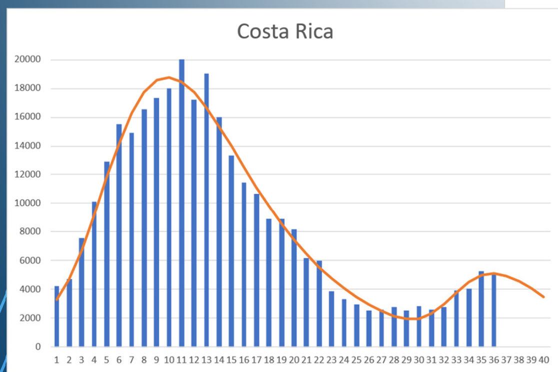
Panamá

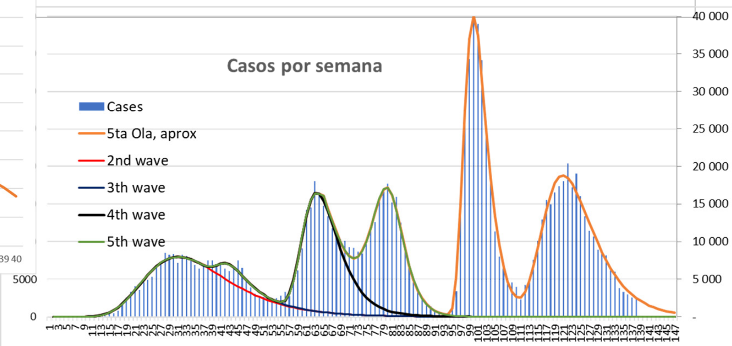


SECOND WAVES AND MORE

Costa Rica

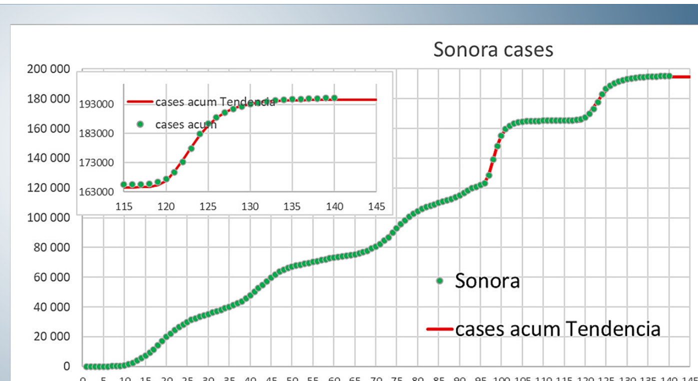


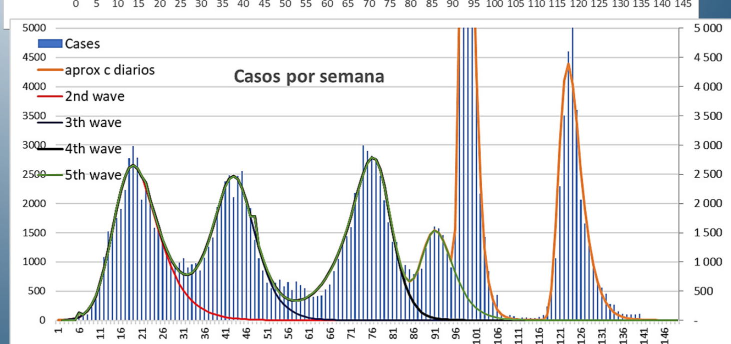




SECOND WAVES AND MORE

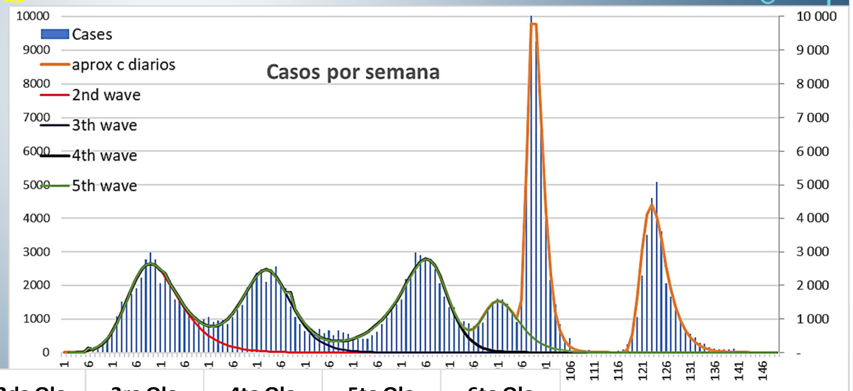
Sonora, México





SECOND WAVES AND MORE

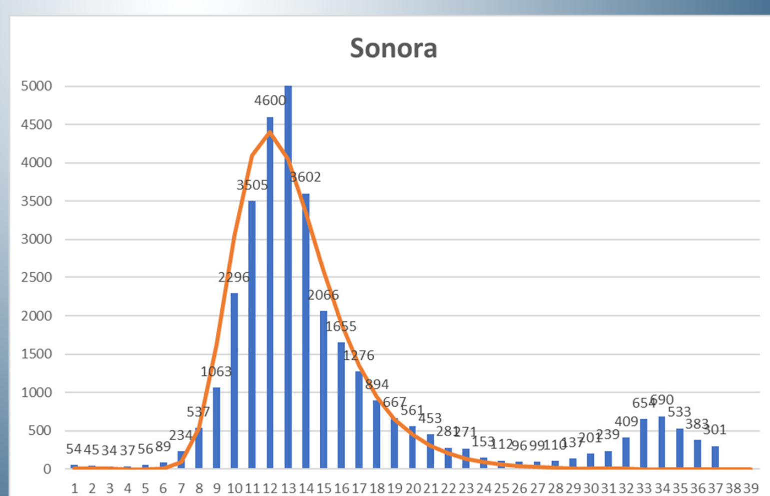
Sonora, México



		1ra Ola	2da Ola	3ra Ola	4ta Ola	5ta Ola	6ta Ola
$M =$	30 004	**37 314**	33 306	39 669	14 987	39 315	30 004
$a =$	-0.401655424	**-0.224331**	-0.296382	-0.403890	-0.313481	-0.701183	-0.401655424
$b =$	-0.074022652	**3.007576**	4.644027	11.308579	1.541733	-1.201003	-0.074022652
$1/c =$	0.00086600	**0.323840**	1.039866	2.45437778	0.29912217	0.00459474	0.000866001
$c =$	1154.733062	**3.09**	0.961662	0.407435	3.343116	217.640048	1154.733062
$t=$	105	**0.0000**	27	50	82	92	105
$\sum(Cr - Cc)^2$	11 477	**632.17**	1 295.96	7 730.72	631.64	399.47	11477.32582
$R^2 =$	0.99998249	0.99970792	0.99983947	0.99994984	0.99995139	0.99995139	0.99998

SECOND WAVES AND MORE

Sonora, México

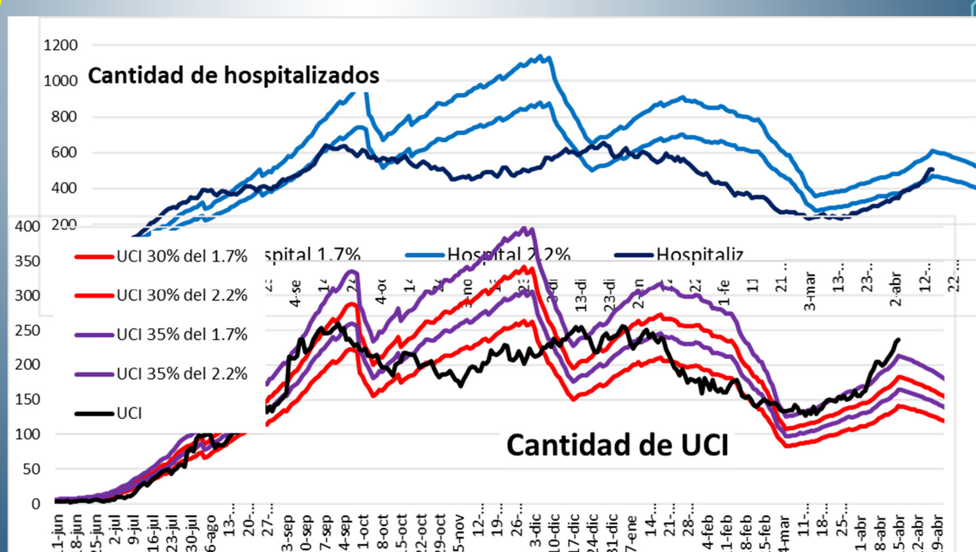


Costa Rica

OTROS DATOS

CURRENT FORECASTS OR TRENDS

Costa Rica



CURRENT FORECASTS OR TRENDS

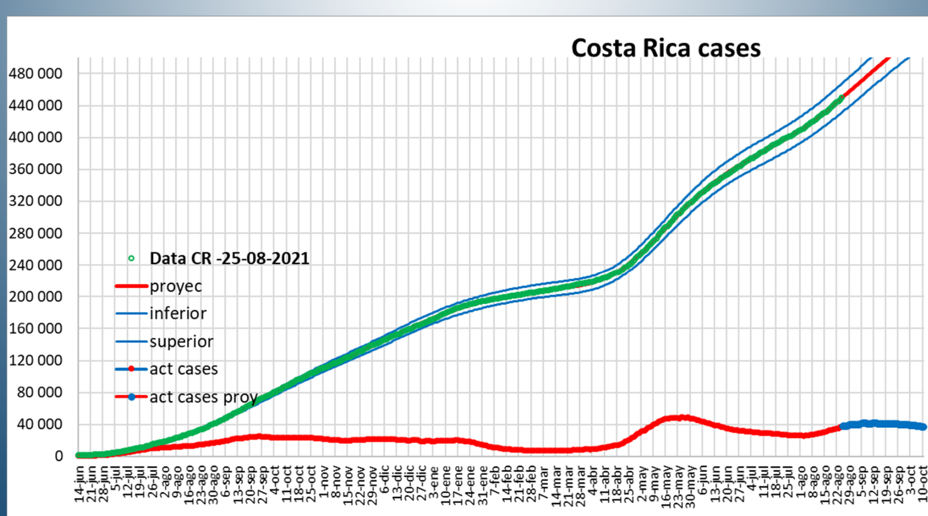
Costa Rica

								alpha 90% 1.645	
	Max	Mean	Min d	Max c	daily n	s	C	Var. Ind.	Confidence interval
%Hospital/ activos	10.29%	2.27%	0.96%	2.43%	**1.59%**	0.414%		0.260	[0.91% , 2.27%]
%ICU / Hospital	73.68%	38.34%	34.98%	59.49%	**44.99%**	6.659%		0.148	[34.0% , 55.9%]
%deads/ cerrados	25.00%	4.62%	1.49%	1.70%	**1.59%**	0.068%		0.043	[1.48% , 1.70%]
%deadsd /Hospital	11.76%	1.92%	0.37%	3.65%	**2.01%**	0.689%		0.342	[0.88% , 3.15%]
%deads d /ICU	66.67%	5.62%	0.67%	9.62%	**4.61%**	1.755%		0.381	[1.72% , 7.49%]
%deads d /Cases	4.23%	1.39%	0.21%	4.23%	**1.48%**	0.738%		0.500	[0.26% , 2.69%]
%ICU / actives	3.22%	0.82%	0.42%	0.92%	**0.70%**	0.141%		0.202	[0.47% , 0.93%]

CURRENT FORECASTS OR TRENDS

Costa Rica "real active cases"

Taking 21 days of daily reported cases



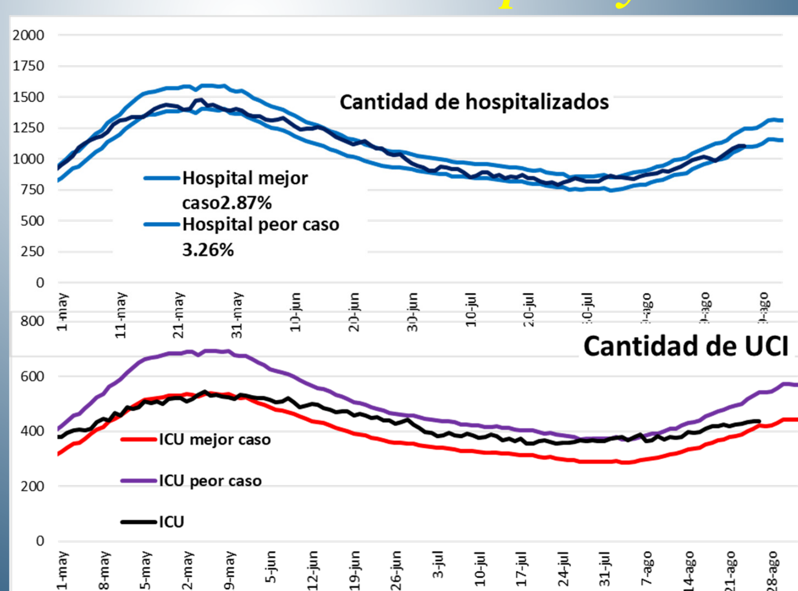
CURRENT FORECASTS OR TRENDS

Costa Rica

	daily n	$\sigma =$	I	Ind. Va	Intervalo de confianza
%Hospital/ actives	1.44%	0.29%		0.203	[1.02% , 1.86%]
%ICU / Hospital	14.26%	0.82%		0.057	[13.1% , 15.4%]
%dead/ closed	0.99%	0.00%		0.001	[0.99% , 1.00%]
%dead /Hospital	1.13%	0.42%		0.375	[0.52% , 1.74%]
%dead d /ICU	7.86%	2.83%		0.360	[3.79% , 11.93%]
%dead d /num dia	0.30%	0.14%		0.464	[0.10% , 0.50%]
%ICU / actives	0.21%	0.04%		0.203	[0.15% , 0.27%]
%dead/ closed-45 days	0.50%	0.06%		0.116	[0.42% , 0.58%]

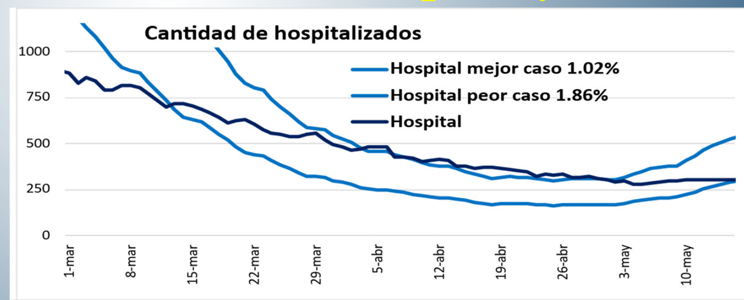
CURRENT FORECASTS OR TRENDS

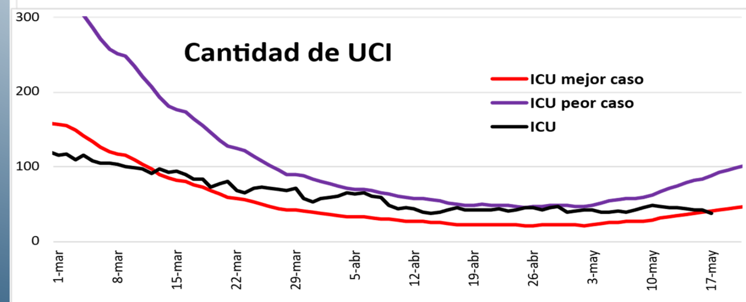
Costa Rica Hospital y ICU



CURRENT FORECASTS OR TRENDS

Costa Rica Hospital y ICU

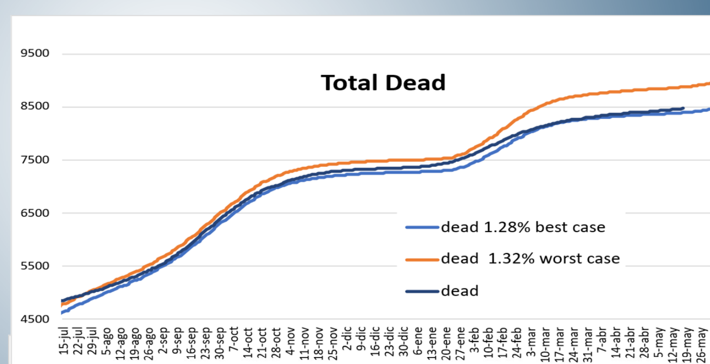


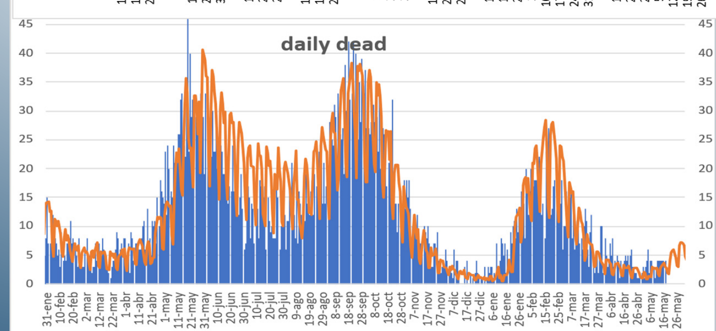


CURRENT FORECASTS OR TRENDS

Costa Rica

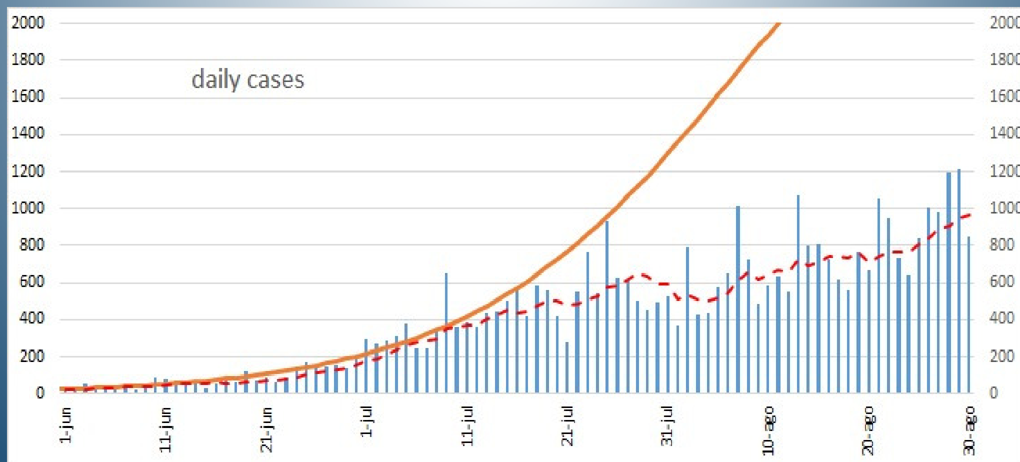
Dead





CURRENT FORECASTS OR TRENDS

Some months ago, the "hammer blow"

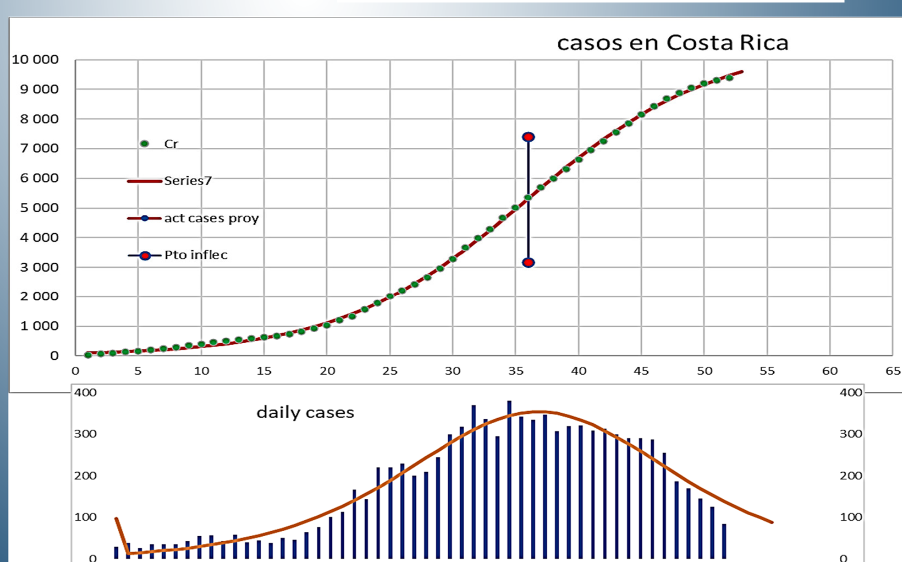


OTHER DISEASES

$$P(t) = \frac{M}{(1 + e^{-at+b})^{\alpha}}$$

Costa Rica

DENGE



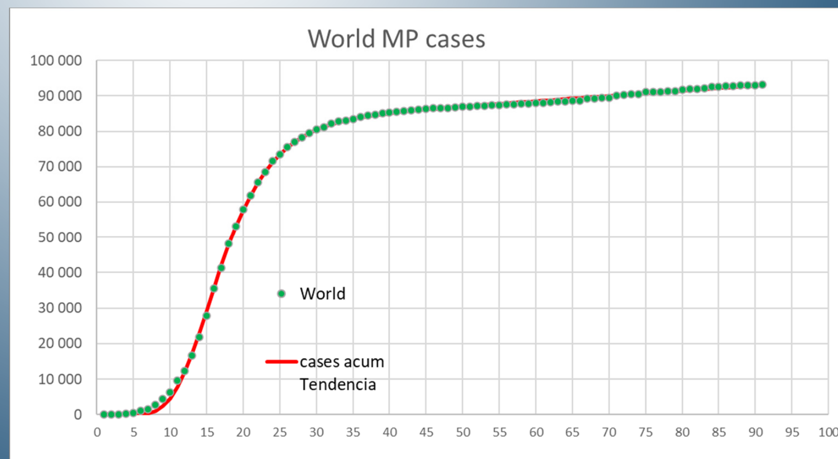
OTHER DISEASES

World

Monkey Pots

$$P(t) = \frac{M + d\,t}{(1 + e^{-at+b})^{c}}$$

M =	79 708
a =	-0.206979263
b =	-6.842752145
1/c =	0.00004686
d=	146.73171320
c =	21339.578984

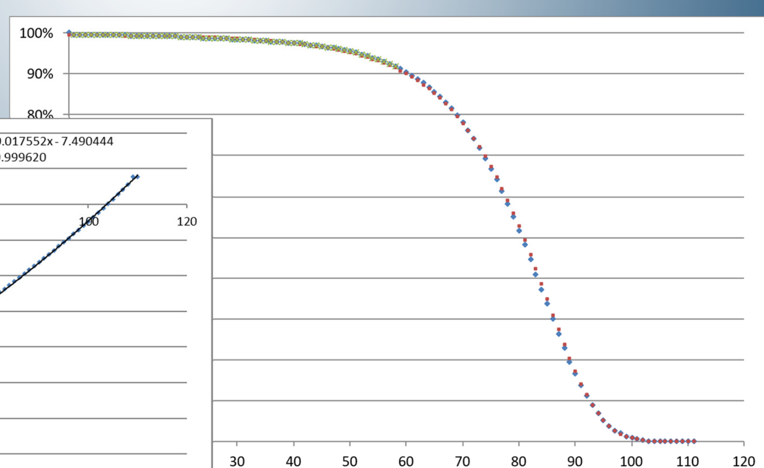


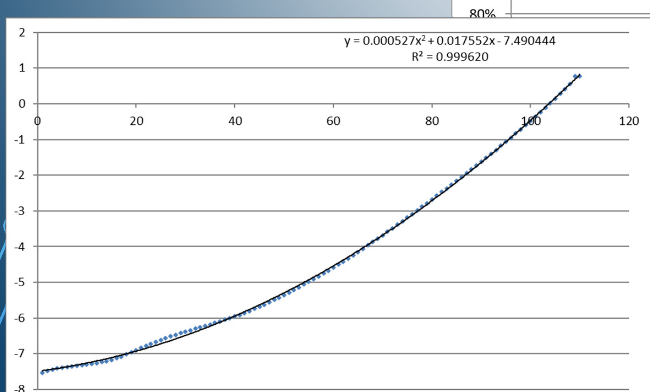
OTROS USOS

Curva de supervivencia

Actuariado

$$P(t) = \frac{M}{(1 + e^{-at^2+bt+c})^{\alpha}}$$





CONCLUSIONS:

- Este método se puede utilizar para ver la tendencia de las poblaciones, en este caso de personas contagiadas por Covid-19, Dengue, Viruela del Mono,
- y podrían ayudar a los expertos en pandemias a tomar las medidas necesarias.
- This method can be used to see the trend of the populations, in this case of people infected by Covid - 19, Monkey pots
- and could help pandemic experts take the necessary action..

FUTURE WORK:

- *Other regions and countries*
- *More studies to refine the results of this work*
- *Using similar curves on other types of data.*
- *Use another software*
- *Improve the algorithm to automate*

- *Otras regiones y países*
- *Más estudios para refinar los resultados de este trabajo*
- *Utilizar curvas similares en otros tipos de datos.*
- *Utilice otro software*
- *Mejore el algoritmo para automatizar*

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THANKS

GRACIAS