Scientific Program

CIMPA Online Pre-School & Research School Algebraic methods in Topology

El Salvador, December 14-19, 2020 & July 11-22, 2022.

Pre-School Online Courses

Duration: 6 hours each.

Foundations of Homotopy Theory

By *Ingrid Martínez*, Universidad de El Salvador.

Language: Spanish.

Abstract: El minicurso de fundamentos de homotopía se desarrollará en tres sesiones donde se abordarán conceptos y resultados importantes de la teoría de homotopía:

Sesión 1: Homotopía de funciones y grupo fundamental. H- espacios, espacios lazo y suspensiones.

Sesión 2: Construcción de espacios: Cilindros y Conos. Grupos de homotopía y sucesiones de homotopía.

Sesión 3: Propiedades de extensión y levantamiento de homotopía.

Bibliography: *Algebraic Topology from Homotopical Viewpoint*; Aguilar M., Gliter S. & Prieto C.

Introduction to Abelian Groups and Modules

By *Gabriel Chicas*, Universidad de El Salvador.

Language: Spanish.

Abstract: En este minicurso abordaremos elementos básicos de la teoría de grupos abelianos y módulos. Nos enfocaremos en algunos problemas de clasificación: el de módulos finitamente generados sobre un dominio de ideales principales, y más generalmente sobre un dominio de Dedekind. Concluiremos con una discusión acerca de módulos proyectivos sobre un dominio de Dedekind.

Bibliography: *Abstract Algebra, 3rd*; Dummit and Foote. *A Course in Homological Algebra, 2nd*; Hilton and Stammbach.

An introduction to Stable Homotopy Theory

By *Omar Antolín*, Universidad Nacional Autónoma de México.

LANGUAGE: Spanish.

ABSTRACT: Los espectros son una manera concreta de representar teorías de cohomología generalizadas en términos de espacios topológicos y tienen una peculiar naturaleza dual, topológica y algebraica. En este mini-curso daremos una introducción a la teoría de homotopía estable, que es la rama de la topología encargada de estudiar a los espectros. Hablaremos de los teoremas de la teoría de homotopía que son precursores de la teoría estable, de las definiciones de espectros y algunos ejemplos clave, y de cómo su estudio es una generalización del álgebra.

BIBLIOGRAPHY: *Stable Homotopy and Generalised Homology*; Adams, Frank; libro clásico, la parte III del libro es la más relevante. *The stable homotopy category*; Malkiewich, Cary; artículo expositorio disponible en la página web del autor: https://people.math.binghamton.edu/malkiewich/stable.pdf. *Stable algebraic topology, 1945–1966*; May, Peter; artículo sobre la historia de un periodo muy fértil en la teoría de homotopía estable, disponible en la página web del autor: http://www.math.uchicago.edu/ may/PAPERS/history.pdf.

Foundations of Module Theory

By *Sean Sather-Wagstaff*, Clemson University.

LANGUAGE: English.

ABSTRACT: One way to understand a geometric object is to understand its symmetries. This idea translates powerfully to numerous areas of mathematics. In algebra, one way this manifests is through the study of modules: one understands a ring (for instance, arising in geometry or number theory) by understanding the objects it influences. (Maybe think of this like the discovery of Pluto, which was first inferred by astronomers observing *something* affecting other celestial bodies.) In this mini-course, we will present foundational material about rings and modules, in preparation for the follow-up course "Foundations of homological algebra applied to modules."

BIBLIOGRAPHY: *Abstract Algebra, 3ed*; Dummit and Foote. *Algebra*; Hungerford.

School Courses

Duration: 6 hours each.

Topology of function spaces

By *Dimas Tejada* and *Ingrid Martínez*, Universidad de El Salvador.

Language: Spanish.

Abstract: Starting at a basic level, for maps between topological spaces the compact and open topology will be analyzed as well as the exponential law. The concepts of smooth manifolds and maps will be introduced, and the main results on the Weak and Strong Topologies on the Space of C^r-Maps between manifolds will be described. Finally, we will introduce the degree modulo 2 and the Brouwer degree of smooth maps. The course will finish by discussing the celebrated Poincaré-Hopf Theorem.

Bibliography: *Differential Topology*; Hirsch, Morris. *An introduction to differentiable manifolds and Riemannian Geometry*; Boothby, William.

Categories and modules

By *Nadia Romero*, Universidad de Guanajuato.

Language: Spanish.

Abstract: We will start by recalling the basic notions and results of category theory, in particular those concerning the category of modules over a ring. Then we will apply these results to the theory of representations of finite groups and finally we will see some recent results in representation theory that make use of these categorical methods.

Bibliography: *Rings and categories of modules*; Anderson, F. y Fuller, K. *Toposes, triples and theories*; Barr M. y Wells C. *Categories for the working mathematician*; Mac Lane, S. *Advanced modern algebra*; Rotman, J.

Algebraic topology from a homotopical viewpoint

By *Alejandra Trujillo*, Centro de Investigación en Matemáticas A.C.

LANGUAGE: Spanish.

ABSTRACT: The objective of this course is to define homology and cohomology groups using Moore and Eilenberg-MacLane spaces. The study subjects are: function spaces, homotopy classes, homotopy groups, fibrations, cofibrations, CW complexes, Moore and de Eilenberg-MacLane spaces, and finally homology and cohomology groups.

BIBLIOGRAPHY: *Algebraic Topology from a Homotopical Viewpoint*; Prieto, Carlos; Aguilar, Marcelo; Gitler, Samuel.

The ∞-categorical point of view for Thom spectra

By *Omar Antolín*, Universidad Nacional Autónoma de México.

LANGUAGE: English.

ABSTRACT: I will describe the ∞-categorical approach developed by Ando, Blumberg, Gepner, Hopkins, and Rezk to Thom spectra of stable spherical fibrations. I will also mention joint work with Tobias Barthel that uses this approach to study the multiplicative structure of such spectra. This multiplicative structure is characterized by a universal property, which is more useful than might seem given its simplicity. As an application we will discuss E_n-orientations and the Thom isomorphism.
I won't assume familiarity with either ∞-categories or with E_n-ring spectra; on the contrary, I hope this mini course will serve as an introduction to these topics.

Foundations of homological algebra applied to modules

By *Sean Sather-Wagstaff*, Clemson University.

LANGUAGE: English.

ABSTRACT: Each module M over a commutative local Noetherian ring R comes equipped with certain numerical invariants, defined homologically, that allow us to detect certain structural information about the module. For example, the projective dimension detects how close M is to being projective. Not only that, but projective dimension of modules can detect whether a ring is regular, as Auslander, Buchsbaum, and Serre proved in the 1950s, and this led to the solution of a famous open question about the localization properties for regular rings. In this course, I will discuss some of these ideas (there are many out there) focusing on a few favorites, including foundational topics.

BIBLIOGRAPHY: *Abstract Algebra, 3ed*; Dummit and Foote (sections 7.1-9.5, 10.1-11.4, 12.1, 13.1-13.2, and pp. 656-657, 706-709, 714-715, 718-719. *Algebra*; Hungerford (Chapters 1-2 and sections V.1, VII.1, VIII.1, and Theorem VIII.4.9). *Introduction to Commutative Algebra*; Atiyah and MacDonald (Chapters 1-3 and 6).

Algebraic K-Theory and connections to Topology

By *Mohamed Elhamdadi*, University of South Florida.

LANGUAGE: English.

ABSTRACT: We will give a foundational course on algebraic K-Theory. We will introduce the K-groups starting from the Grothendieck group K_0 to higher K-groups including the Plus-construction of Quillen. Connections to Hochschild and cyclic homology will be given. If time allows we will discuss a variation of K-theory called Karoubi's K-theory.

BIBLIOGRAPHY: *The K-book. An introduction to algebraic K-theory*; Weibel, Charles. *Cyclic homology*; Loday, Jean-Louis. *Algebraic K-theory and its applications*; Rosenberg, Jonathan. *Approach to algebraic K-theory. Research Notes in Mathematics, 56*; Berrick, A. Jon.

School Talks

Duration: 1 hour each.

Commutativity of Lie groups

By *Omar Antolín*, Universidad Nacional Autónoma de México.

Abstract: Given a Lie group G, we can think of the space of homomorphisms $\mathrm{Hom}(Z^n, G)$ as the space of n-tuples of elements of G that commute pairwise. These spaces are more subtle than one might think, and even basic invariants such as the number of connected components can lead to surprising results. Fixing G and varying n we can construct what is known as the classifying space for commutativity in G. I will describe what is known about these classifying spaces, whose study has just begun.

Complete Intersection Injective Dimensions

By *Sean Sather-Wagstaff*, Clemson University.

Abstract: The complete intersection dimension of a module M over a commutative local Noetherian ring R is a homological invariant that detects many properties like projective resolution properties of modules over formal complete intersection rings. It was introduced by Avramov, Gasharov, and Peeva in 1997. In this talk, we will introduce a new variant of this, the Hom complete intersection dimension (Hom-CI-id for short), that detects many properties like injective resolution properties of modules over formal complete intersection rings. For instance, every module over a formal complete intersection ring has finite Hom-CI-id; conversely, if the residue field has finite Hom-CI-id, then the ring is a formal complete intersection. Also, the Hom-CI-id satisfies versions of the Bass formula, the Chouinard formula, and Bass' conjecture. This is joint work with Jon Totushek.

Some non-associative algebraic structures in Low dimensional topology

By *Mohamed Elhamdadi*, University of South Florida.

Abstract: We will introduce some non-associative algebraic structures called racks and motivated by the study of knots in the 3-space and knotted surfaces in 4-space. We will define a cohomology theory for these structures and will explain its use in constructing invariants of knots. We will explain how these structures give an enhancement when considered in the category of topological spaces or Lie groups.

Organizers

Date of issue: December, 2020.